Engineering Case Library

ECL 103

Space Crank Application

Designers at Mark Systems Incorporated ran into a vexing mechanical design problem which involved moving a lens into and out of an optical plane. They suggested a number of possible solutions, none of which satisfied the project engineer. The author of this case study was called in as a consultant; details of the problem and of his solution to it are documented on the following pages.

Prepared by Mr. Aaron Baumgarten with support from the National Science Foundation through the Case Program, Design Division, Department of Mechanical Engineering.

Mark Systems Incorporated is a young Santa Clara company whose principal business has been the manufacture of portable film processing equipment. In order to insure the company's continuing existence and growth, the management of Mark Systems has been eager to develop other products.

Early in 1966 MSI management learned of a unique engineering concept which solved in principle a difficult problem in the field of optical equipment. A good, practical conversion of this solution to functioning hardware would be of great interest to both civilian and military users. The marketing people at MSI conducted a survey intended to gauge the total market. Their estimates of the total market potential were so great that even after allowing for normal "Salesman's Enthusiasm" MSI management felt justified in making the decision to develop a working model for demonstration purposes.

The device chosen to demonstrate the basic concept was a special binocular; it was a relatively crude device, but it sufficed to show that the basic idea was a good one. Proof of the effectiveness of the demonstrator was provided by the fact that MSI began receiving orders for production units. Undoubtedly, among the users of the production unit would be sportsmen and military people holding the device in their hands while working under adverse field conditions.

A group of MSI designers was assigned the task of developing from the demonstration model a design suitable for production; they quickly ran into several nasty mechanical design problems. One of these was the problem of moving a lens into and out of an optical train by the use of the left thumb or a finger squeeze of the left hand, possibly while wearing heavy gloves (see Exhibit 1 for a visualization of this problem). The project engineer for this contract was young, aggressive, creative, and not easily pleased with marginal solutions. His designers offered slotted links, cable drives, bell cranks, spur and bevel gear trains, and cams (see Exhibits 1,2,3), but none of these satisfied him.

His objections were that there were too many pieces (expensive); or too much friction and weight (a military man, for example, would throw the device away if it were difficult to use); or too delicate or not positive (reliability is important!). He kept reminding his designers of the design criteria he had established for this particular problem. The mechanism had to:

1. have as few parts as possible
2. be light weight and compact
3. be positive in its action
4. be smooth acting and free of jamming (easily operable to avoid operator fatigue)
5. be relatively insensitive to shock
6. be inexpensive to fabricate
7. have as little wear as possible

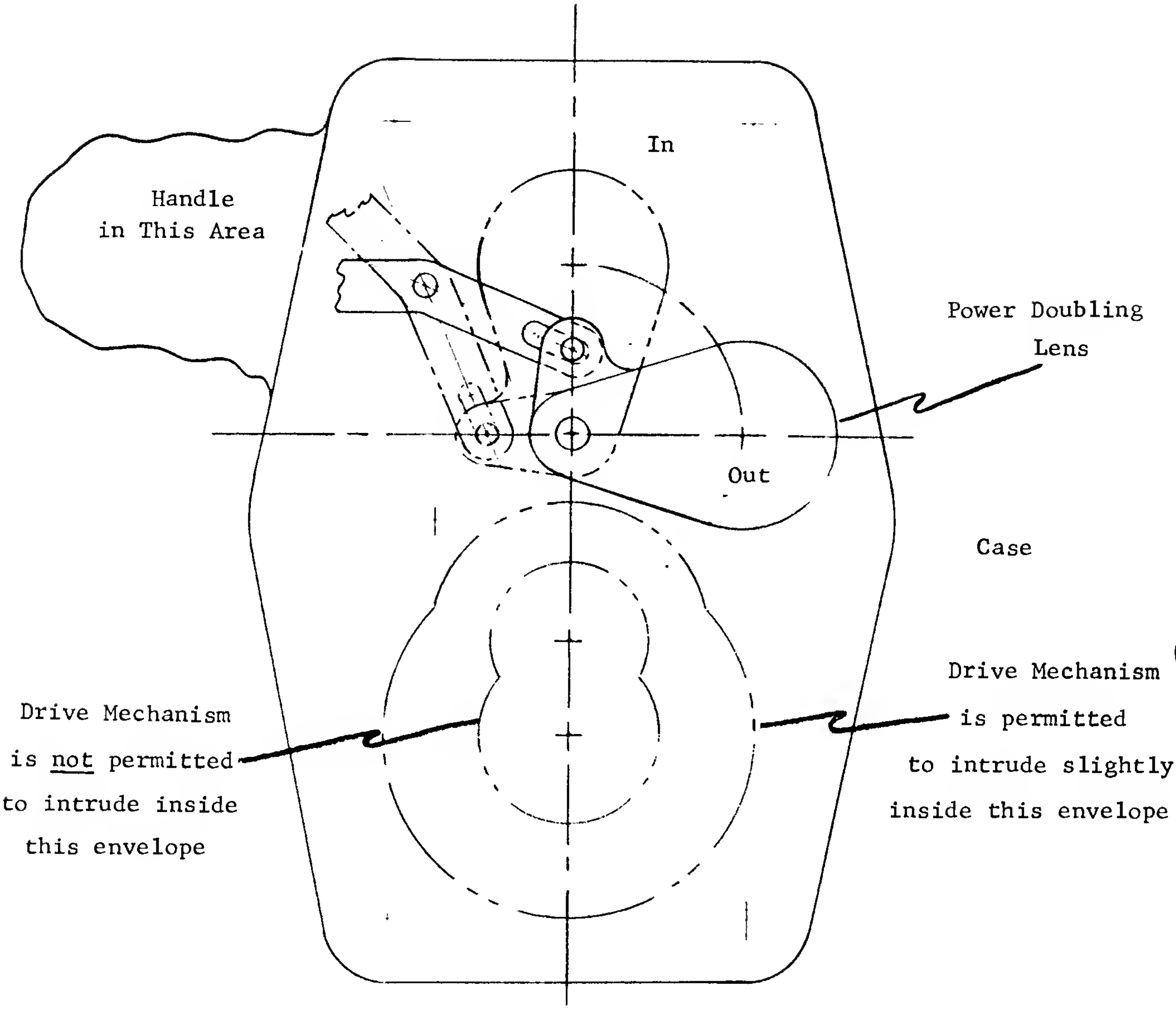
The designers replied that after all the ideas they had discussed, there didn't seem to be anything else to try. With an eye on his schedule and budget, the project engineer decided to invest in some outside consulting help from the author of this case history. The decision was based on the fact that the author had participated in the very early work on the demonstrator and was not new to its problems. Also, all of the designers knew him and would accept him readily. Twenty working hours were allowed to present a feasible concept. The author listened to the story and commented, "Unfortunately, I can agree with all of you. None of the proposed solutions looks good, but what else is there to try?" The project engineer's answer was, "Don't <u>ask</u> us, you're being paid to <u>tell</u> us what else there is to try." After an embarassing silence, the author said, "Wait a minute. I'm not certain, but there may be something. I remember a mechanism called a space crank - some of the modern text books on kinematics describe it, and if my memory serves me correctly there was an article in a trade journal some time ago which discussed this mechanism. I've never used it before but I think it holds a lot of promise for your application. I'll look through my file of old magazine articles to see if I can find something on it." The search did turn up an article on the subject of space cranks. From the description, it seemed to fit the operating and fabricating requirements quite well. For example, given a

uniform input, the linkage output characteristics (displacement, velocity, and acceleration) were quite smooth and free of discontinuities. This indicated smooth action which would be free of jamming. The displacement curve, in particular, displayed a very interesting and desirable shape; in the region of extreme angular displacement this curve became quite flat in a manner somewhat like a "dwell". This in turn meant that it would not be necessary to develop a precise input motion in order to achieve a reasonably precise output motion. Furthermore, in this dwell condition the linkage formed a natural positive acting detent, i.e., it could not be driven by its load. Also, the manufacture of links joined by pins is relatively simple and inexpensive (especially inexpensive when considering production quantities which might number in the thousands. In terms of cost the closest competitor would be catalog gears, but even they would cost more than the links.) It seemed as though the links could be small (i.e., light) and yet rugged (i.e., dependable and insensitive to shock). For these reasons a layout was prepared which showed that the space linkage approach was indeed feasible as well as attractive from a functional and manufacturing point of view.

However, in order to build a prototype of the space crank so that its performance could be verified, it was necessary to establish the values of the angles which enter into the design equations (see Exhibit 4). Inasmuch as the desired maximum output angle was known (Φ max=90°) and the maximum output angle is a simple function of the angle called gamma (Φ max=2 gamma), part of the problem was already solved. The design equations show, however, that the angle lambda influences the displacement, velocity, and acceleration characteristics of the crank output. The details of this influence were not immediately obvious to the author from an examination of the equations. In addition, some constraint on the range of permissible values was inherent in the space available (see Exhibit 6). This was especially important because of the dwell characteristic of the displacement curves in the region of maximum output angle. The available informtion indicated that a desirable displacement characteristic could be achieved by using small values for the angle lambda; but this information was insufficient for a design decision on a production model.

Somehow the author had to investigate this aspect of the crank design in order to develop a "feeling" for how it would behave. There was nothing available in what remained of his original twenty hours except to attempt a manual solution of the equations for several representative values of lambda and gamma. The calculations were done with the aid of a desk-top calculator (how welcome a computer would have been!) A sample of the format used for this work is shown as Exhibit 5. Curves showing typical results are Exhibit 7.

On the basis of these calculations (and the curves plotted from them), the published curves in the available literature, and the space available in the actual product, values of gamma and lambda were adopted for the production model and the detailed design was started. As of this date, March 1968, the production drawings are almost ready for release. Exhibit 8a to 8d shows the drawings of the actual parts which are to be used for the current production run. Exhibit 9 is a photograph of the links.

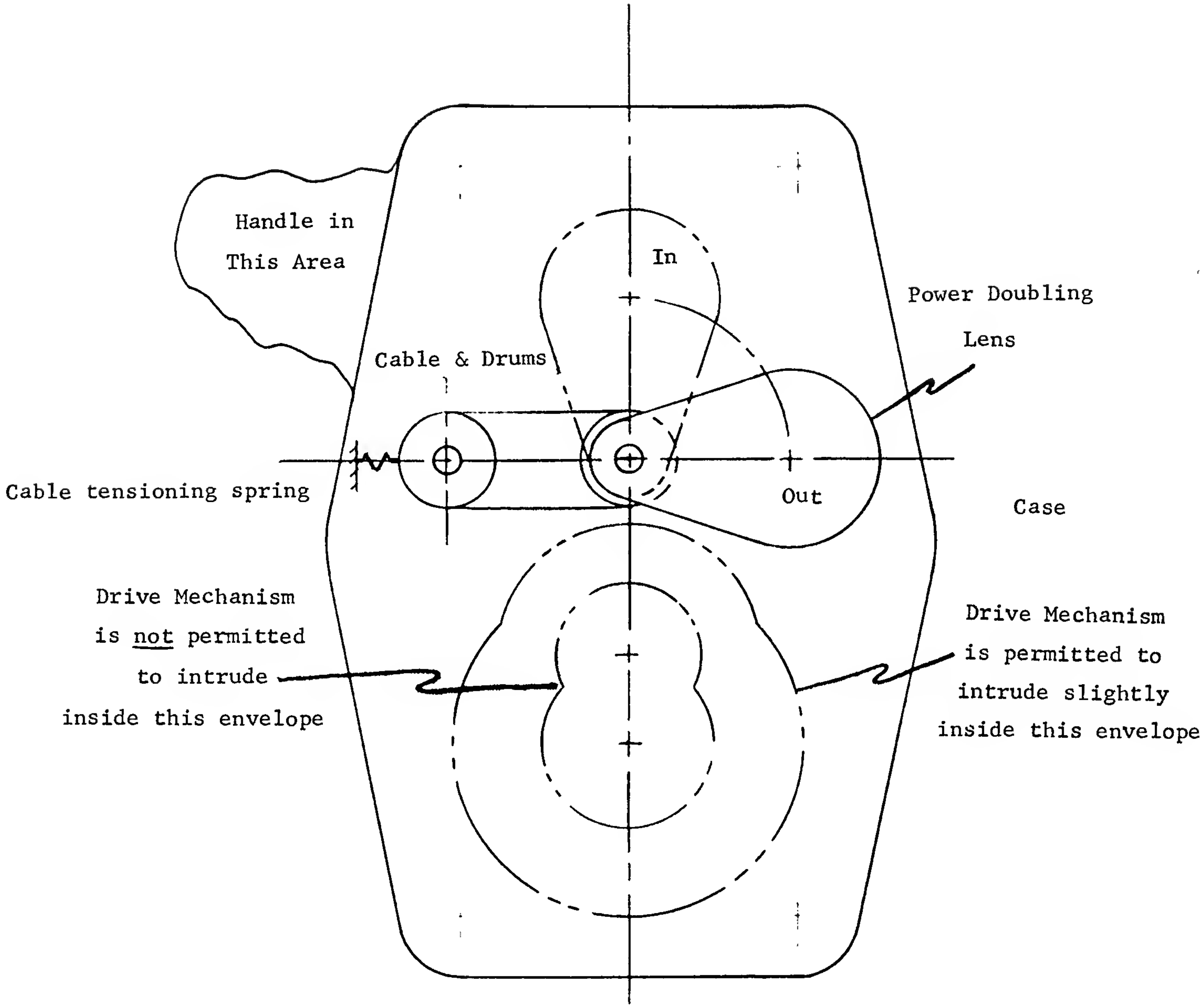


Slotted Bell Crank

1 requires bi-directional movement of input

2 requires detent to lock the lens in "in" position

3 high friction and wear in slot



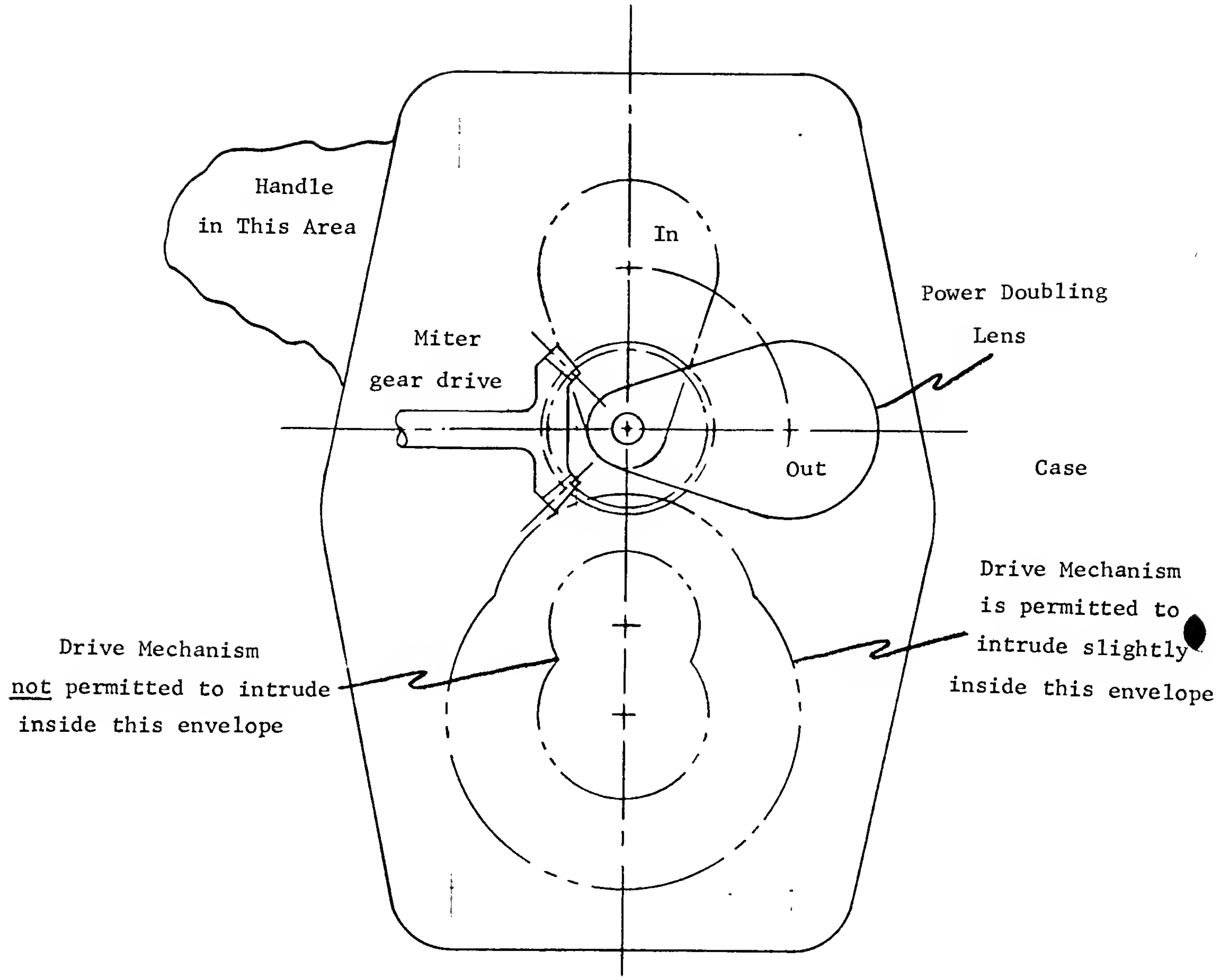
Cable Drive

1 requires bi-directional movement of input crank

2 requires detent to lock the lens in "in" position

3 bulky & expensive

4 cable is susceptible to breakage

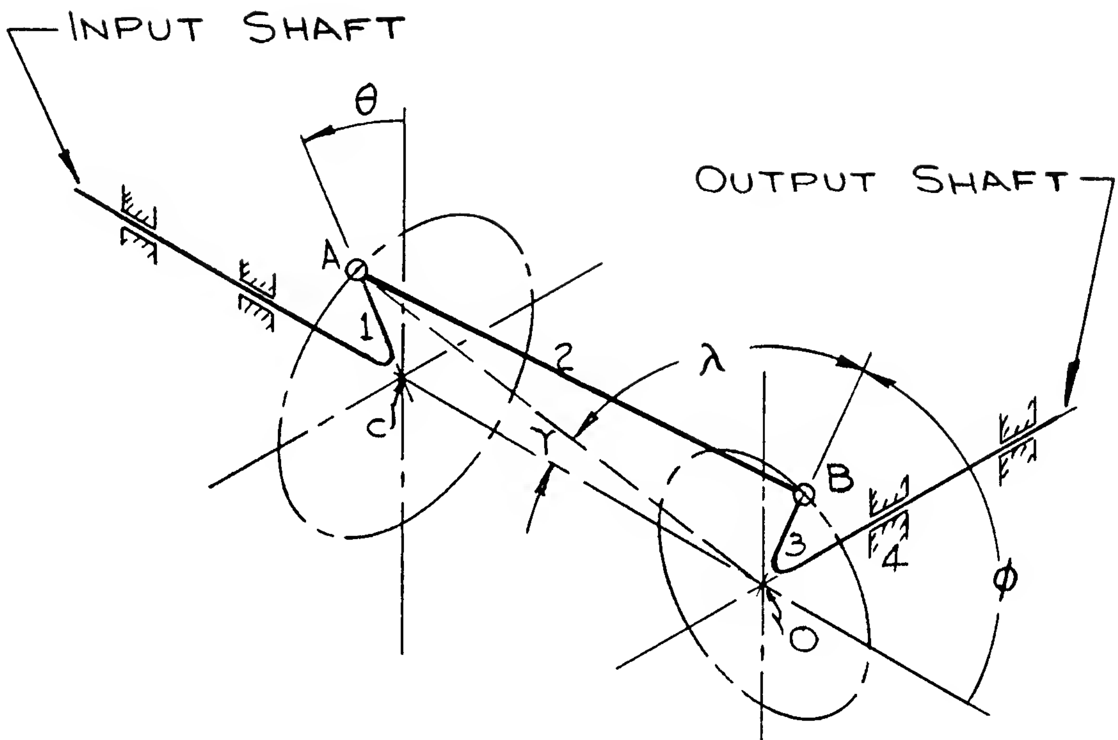


Gear Drive

1 requires bi-directional movement of input

2 requires detent to lock the lens in "in" position

3 bulky & expensive



Schematic of Space Crank

DESIGN EQUATIONS

I. DISPLACEMENT EQUATION

$$\cos\phi = \tan\gamma\cos\theta\ \sin\phi - \frac{\cos\lambda}{\cos\gamma}$$

γ and λ are constant for any given linkage.

II. VELOCITY EQUATION

$$\omega_{output} = \omega_{input}\left(\frac{\tan\gamma\sin\theta}{1 + \tan\gamma\cos\theta\ \cot\phi}\right)$$

III. ACCELERATION EQUATION (For Constant Input Angular Velocity)

$$\alpha_{output} \doteq \frac{\left(\omega_i^2 + \omega_o^2\ \tan\gamma\cos\theta\ +\ \omega_o\cot\phi\left(2\,\omega_i\ \tan\gamma\sin\theta - \omega_o\right)\right.}{1 + \tan\gamma\cos\theta\ \cot\phi}$$

Exhibit 5

$\gamma = 45^\circ$

$a = \tan \gamma = 1.0000$

$\lambda = 75^\circ$

$b = \dfrac{\cos \lambda}{\cos \gamma} = \dfrac{.2588}{.7071} = .3660$

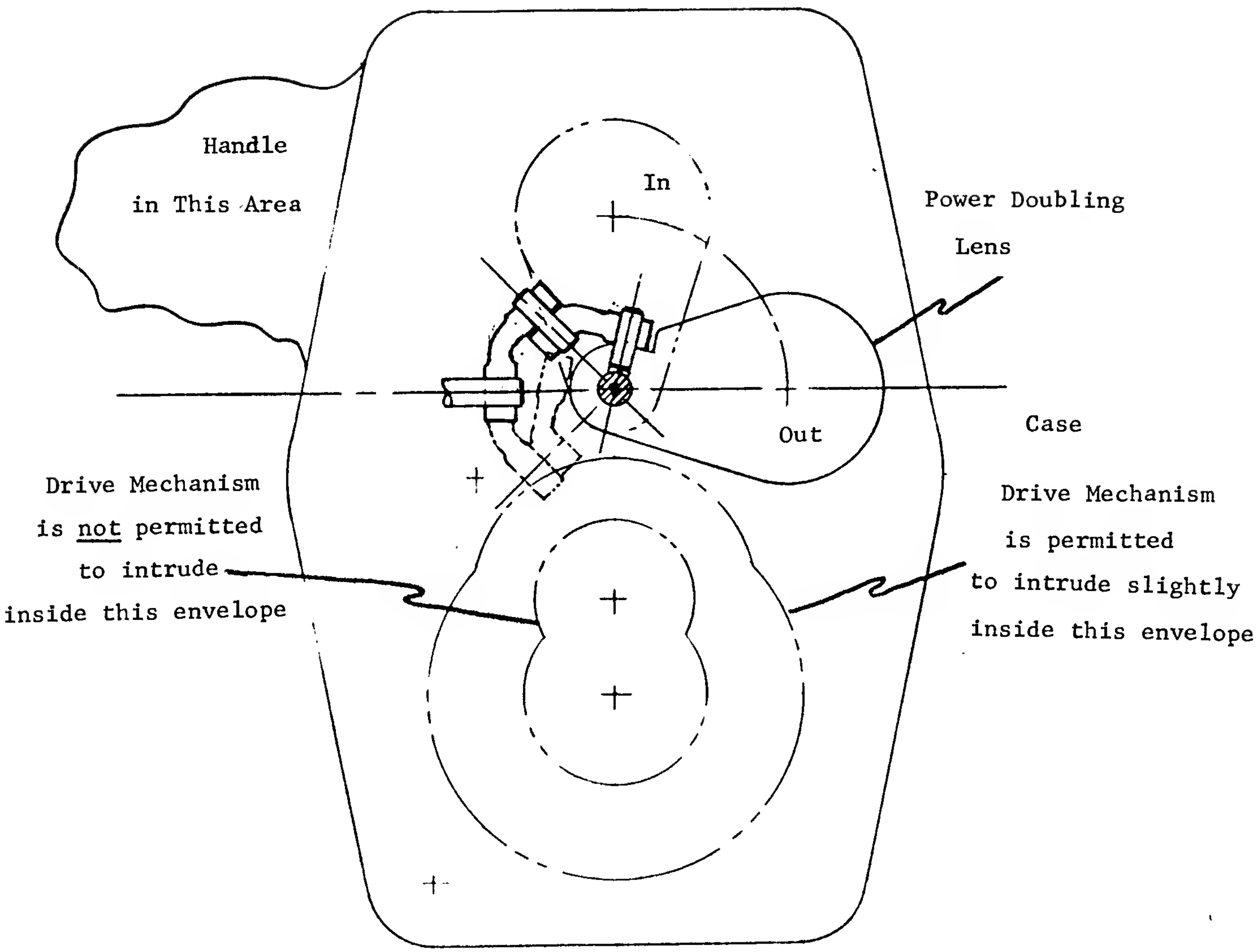
I θ	II $\sin \theta$	III $\cos \theta$	IV $1+a^2\cos^2\theta$	V $\dfrac{b}{1+a^2\cos^2\theta}$	VI $\left[\dfrac{b}{1+a^2\cos^2\theta}\right]^2$	VII $b^2-a^2\cos^2\theta$	VIII $\dfrac{b^2-a^2\cos^2\theta}{1+a^2\cos^2\theta}$	IX $\left[\dfrac{b}{1+a^2\cos^2\theta}\right]^2 - \dfrac{b^2-a^2\cos^2\theta}{1+a^2\cos^2\theta}$	X $\sqrt{}$ preceding
0°	0	1.0000	2.0000	.18300	.03348	-.86604	-.43302	.46650	.6830
15°	.2588	.9659	1.93296	.18934	.03584	-.79900	-.41335	.44919	.6702
30°	.5000	.8572	1.73479	.21097	.04450	-.60083	-.34634	.39084	.6251
45°	.7071	.7071	1.49999	.24400	.05954	-.36603	-.24402	.30356	.5509
60°	.8572	.5000	1.25000	.29280	.08573	-.11604	-.09283	.17856	.422
75°	.9659	.2588	1.06697	.34302	.11766	.06698	.06277	.05489	.234
90°	1.0000	0	1.00000	.36600	.13396	.133956	.133956	.00000	.000
105°	.9659	-.2588	1.06697	.34302	.11766	.06698	.06277	.05489	42
120°	.8572	-.5000	1.25000	.29280	.08573	-.11604	-.09283	.17856	.422
135°	.7071	-.7071	1.49999	.24400	.05954	-.36603	-.24402	.30356	.550
150°	.5000	-.8572	1.73479	.21097	.04450	-.60083	-.34634	.39084	.625
165°	.2588	-.9659	1.93296	.18934	.03584	-.79900	-.41335	.44919	.670
180°	0	-1.000	2.00000	.18300	.03348	-.86604	-.43302	.46650	.683

	cos φ = +col V ÷ col X	φ	Δφ
)	.50000	60°	0
	.48087	61°-16'	1°-16"
	.41420	65°-32'	5°-32'
	.30696	72°-08'	12°-08'
	.12976	82°-33'	22°-33'
	-.10874	96°-14'	36°-14'
o	-.36600	111°-28'	51°-28'
	-.57730	125°-16'	65°-16'
	-.71536	135°-40'	75°-40'
6	-.79496	142°-38'	82°-38'
7	-.83614	146°-44'	86°-44'
1	-.85955	149°-15'	89°-15'
D	-.86600	150°	90°

$\gamma = 45°$ $\lambda = 105°$

$a = \tan\gamma = 1.0000$ $b = \dfrac{\cos\lambda}{\cos\gamma} = \dfrac{-.2588}{.7071} = -.36600$

XIV $\frac{b}{1+a^2\cos^2\theta}$	XV	XVI	XVII	XVIII	XIX	XX cos φ = -col XIV ÷ col XIX	XXI φ
-.18300	IDENTICAL TO COLUMN VI					.8660	30°
-.18934						.85955	30°-45'
-.21097		IDENTICAL TO COLUMN VII				.83614	33°-16'
-.24400			IDENTICAL TO COLUMN VIII			.79496	37°-22'
-.29280				IDENTICAL TO COLUMN IX		.71536	44°-20'
-.34302					IDENTICAL TO COLUMN X	.57730	54°-44'
-.36600						.36600	68°-32'
-.34302						.10874	83°-46'
-.29280						-.12976	[illegible]
-.24400						-.30696	[illegible]
-.21097						-.41420	114°-28'
-.18934						-.48067	118°-44'
-.18300						-.5000	120°



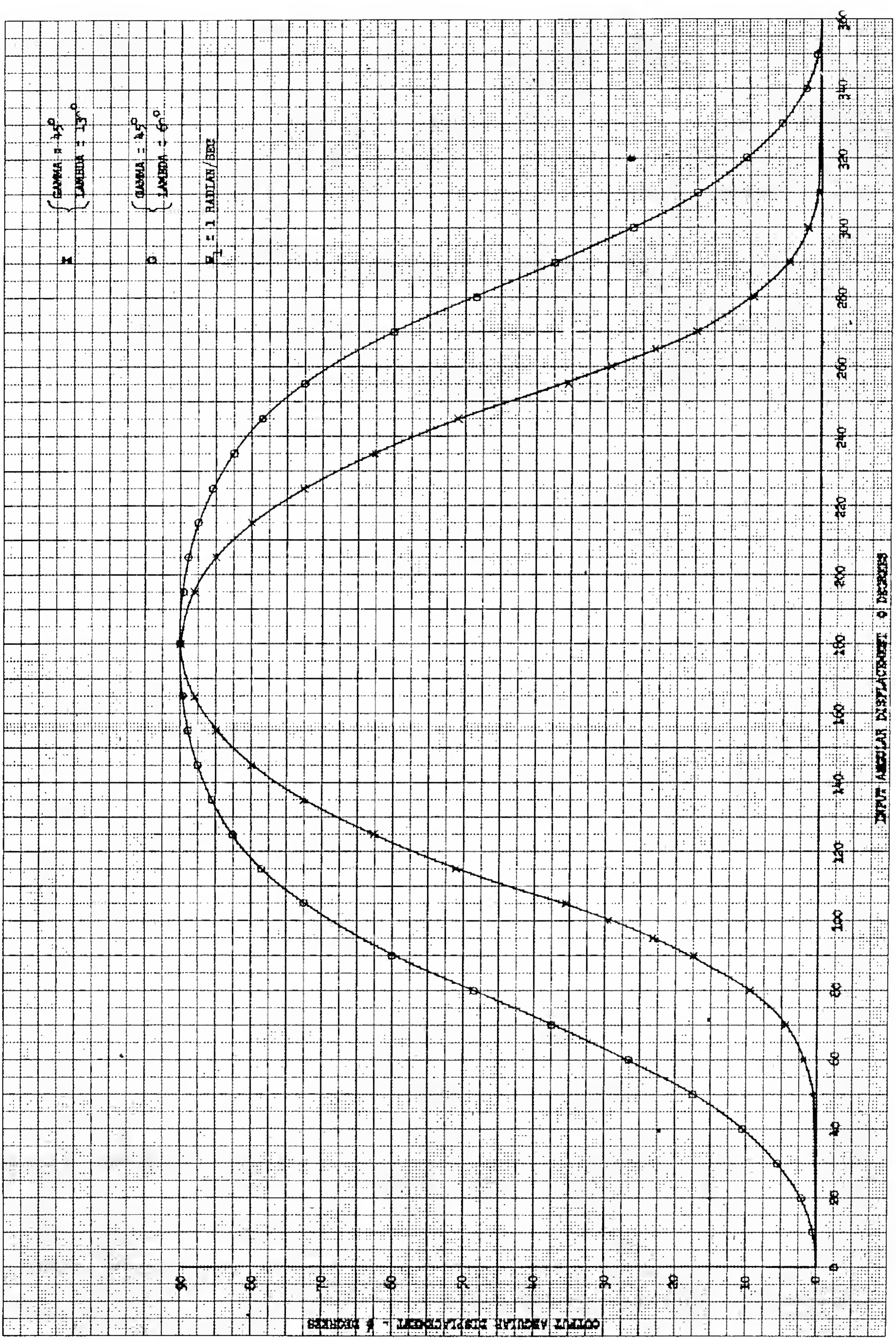
Space Crank

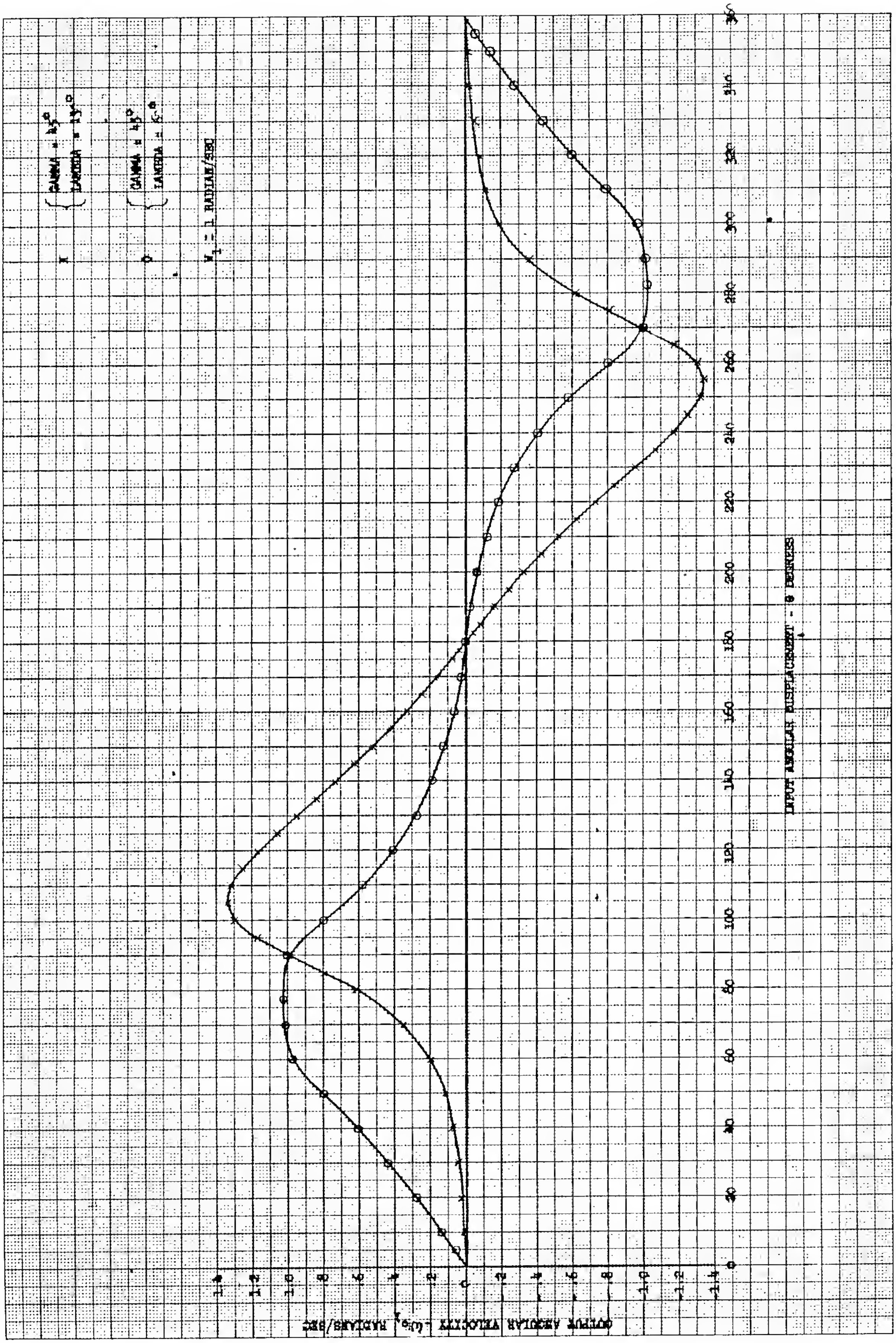
1 requires uni-directional movement on input crank

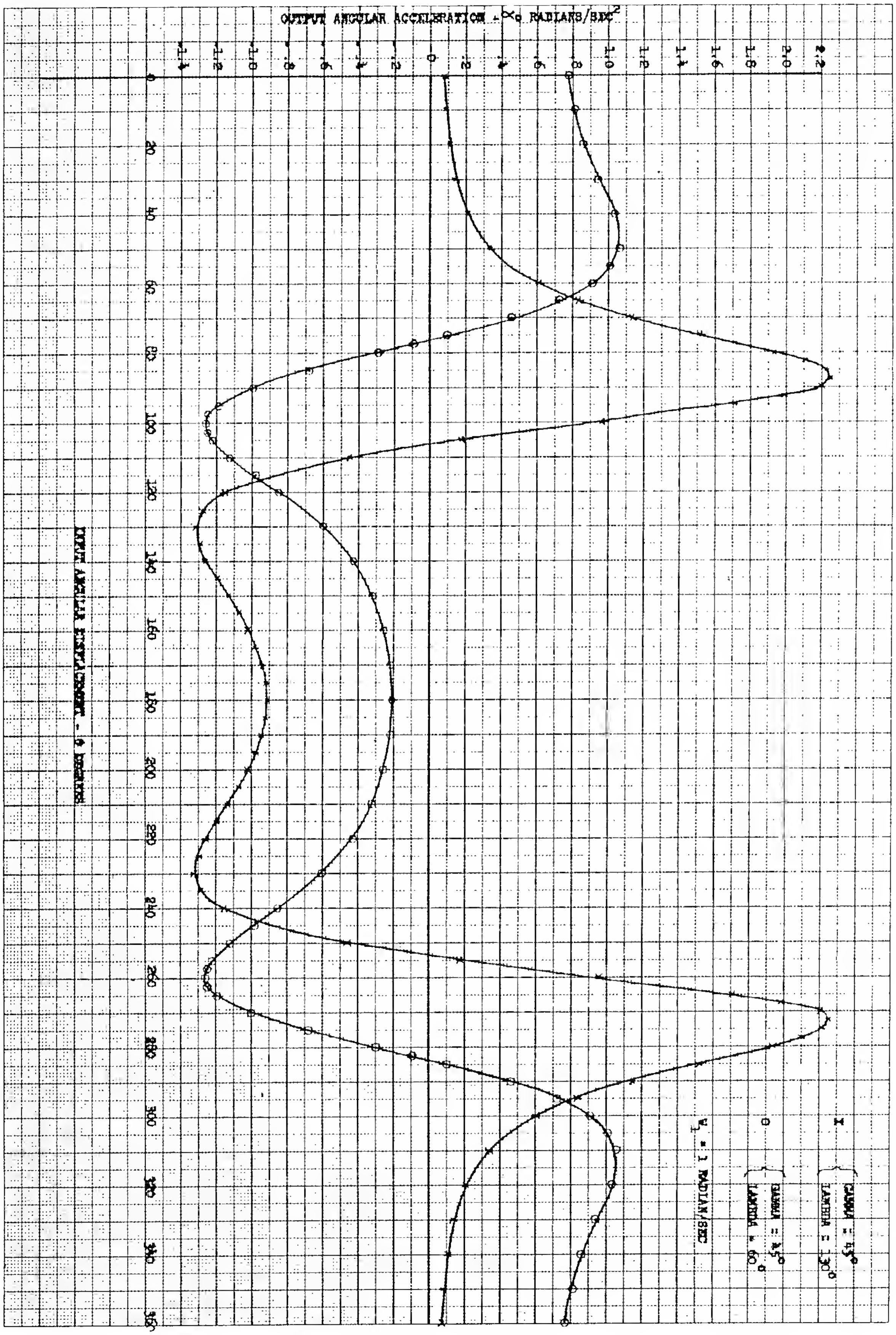
2 linkage "locks" lens in "in" position and does not require an additional detent

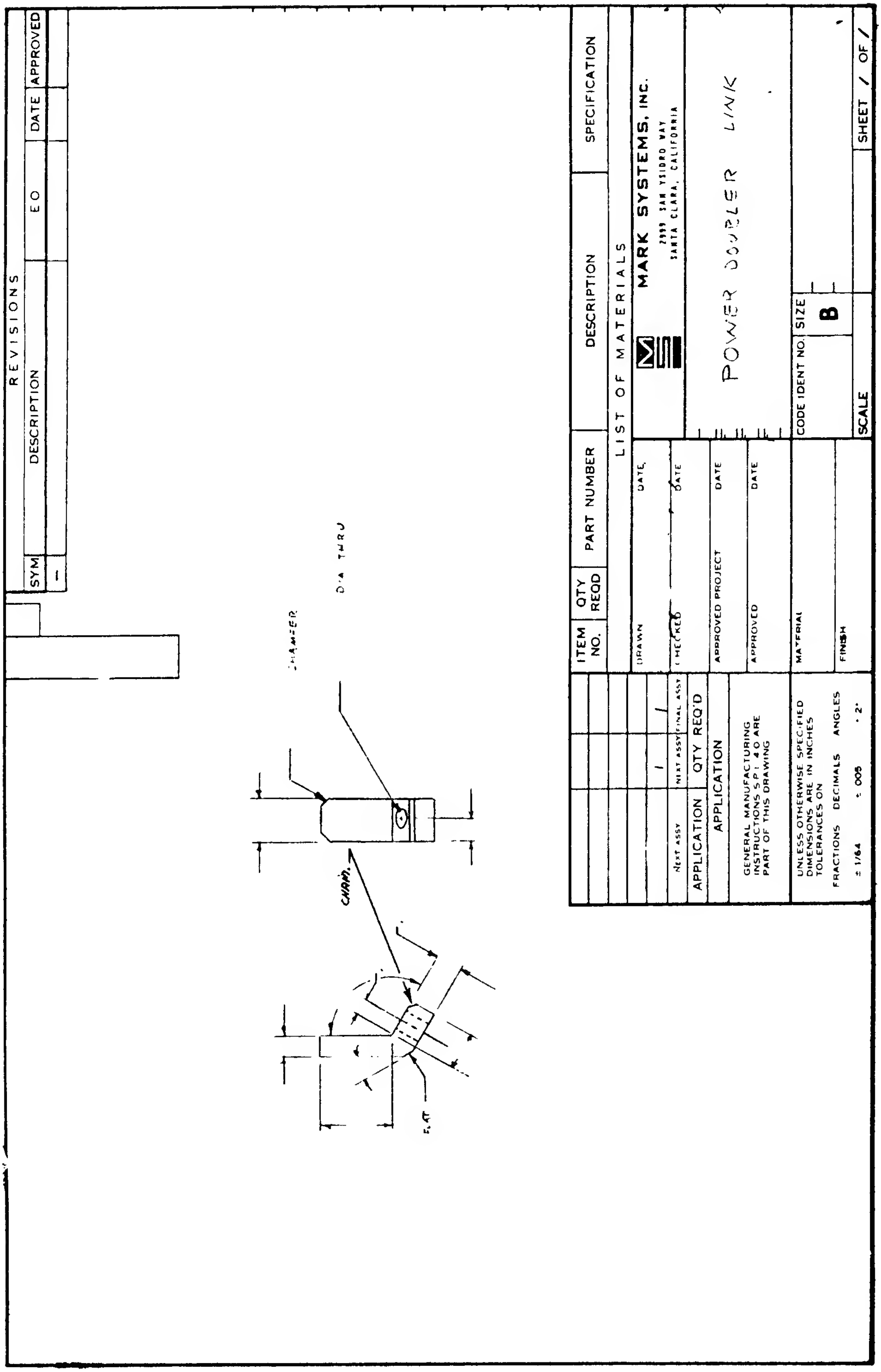
3 pin joints are rugged

4 inexpensive

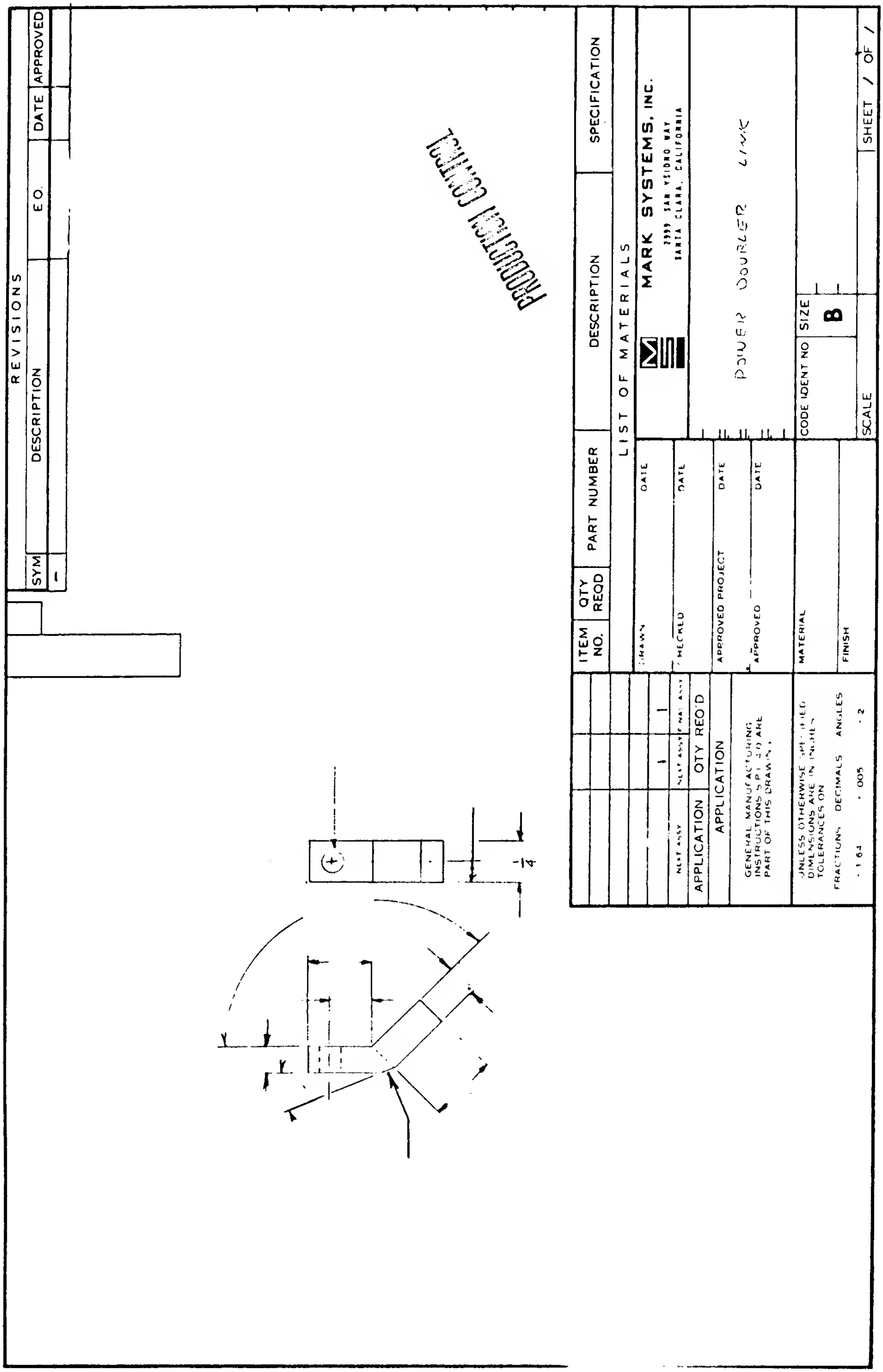

GAMMA = 45°
LAMBDA = 15°
GAMMA = 45°
LAMBDA = 6°
ω_1 = 1 RADIAN/SEC
INPUT ANGULAR DISPLACEMENT θ DEGREES
OUTPUT ANGULAR DISPLACEMENT - ψ DEGREES
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360
10
20
30
40
50
60
70
80
90


x GAMMA = 45°, LAMBDA = 15°
o GAMMA = 45°, LAMBDA = 6°
W_1 = 1 RADIAN/SEC
INPUT ANGULAR DISPLACEMENT - θ DEGREES
OUTPUT ANGULAR VELOCITY - ω, RADIANS/SEC

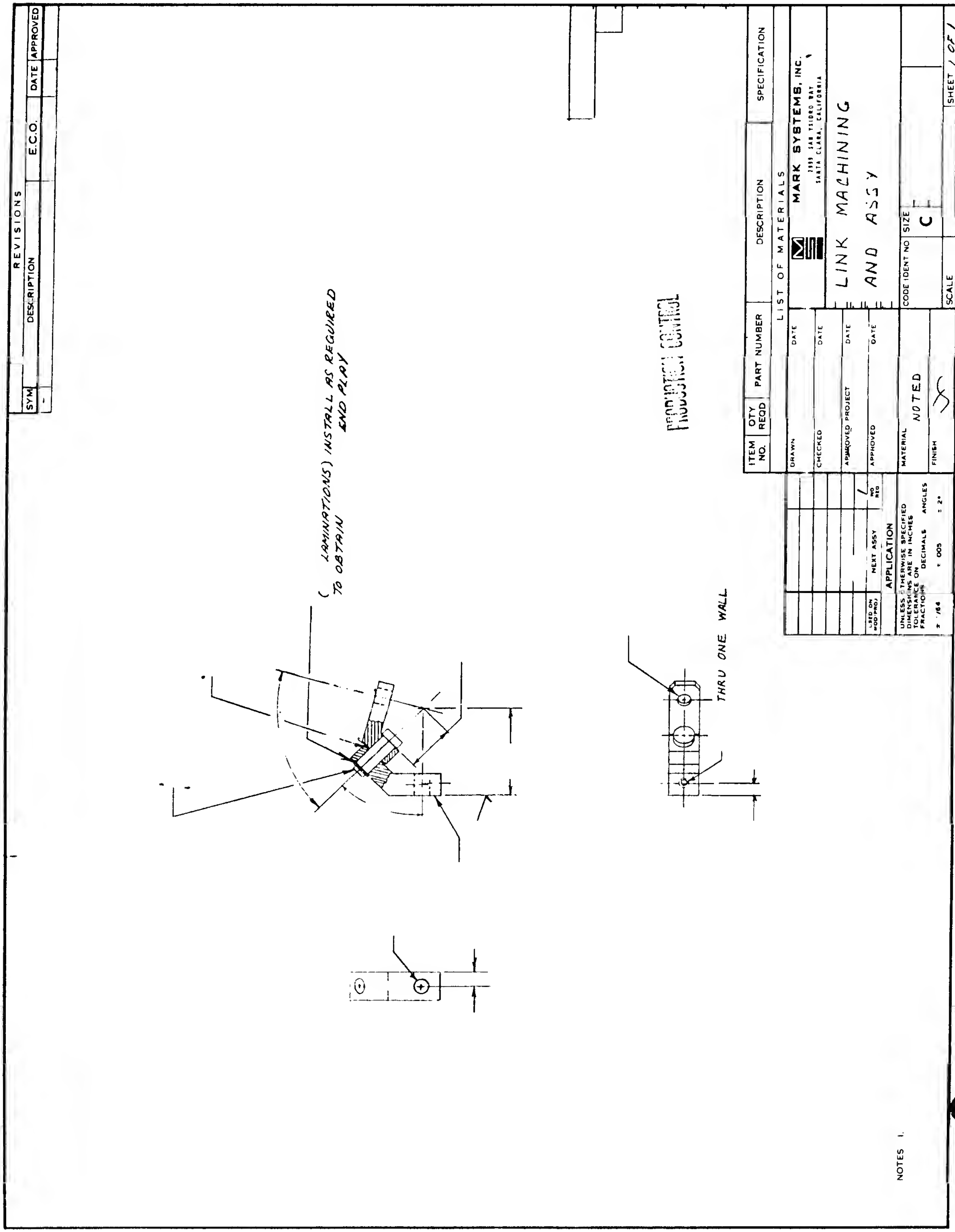

OUTPUT ANGULAR ACCELERATION - α_o RADIANS/SEC^2
INPUT ANGULAR DISPLACEMENT - ϕ DEGREES
GAMMA = 45°
LAMBDA = 30°
GAMMA = 45°
LAMBDA = 60°
W_1 = 1 RADIAN/SEC



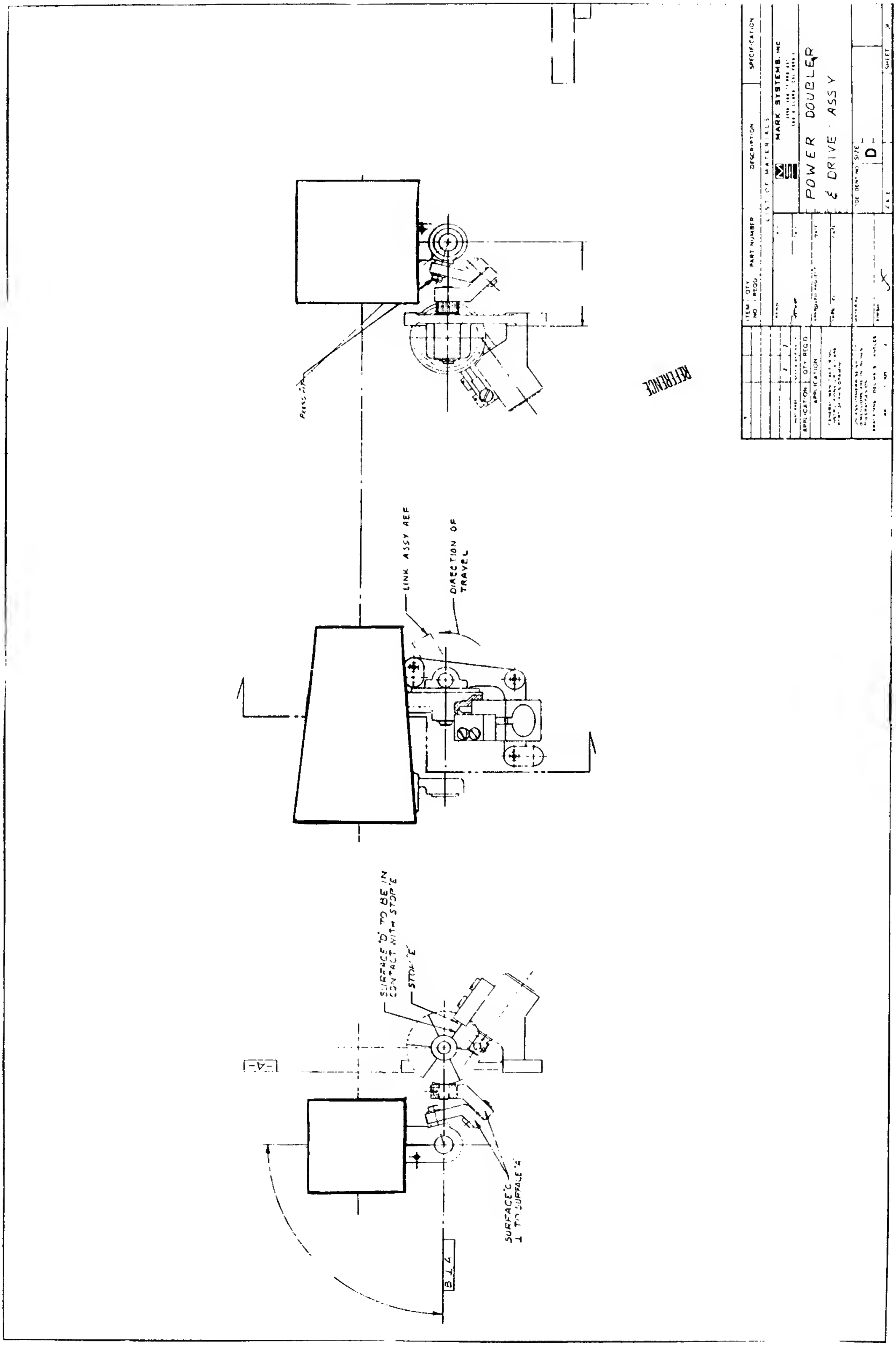
Power Doubler Link



Power Doubler Link



Link Machining and Assembly


MARK SYSTEMS, INC
POWER DOUBLER
¢ DRIVE ASSY
D
LINK ASSY REF
DIRECTION OF TRAVEL
SURFACE 'D' TO BE IN CONTACT WITH STOP 'E'
STOP 'E'
SURFACE 'C' ⊥ TO SURFACE 'A'
REFERENCE
-A-

Photograph of Power Doubler Link Assembly

This case history may be used in both undergraduate as well as graduate courses in kinematics.

The design problem described here affords an opportunity to discuss the following:

1. In solving his problem, the author implicitly sought a mechanism which would produce an oscillatory motion from a uni-directional input motion. Plane four-bar-linkages (i.e., crank-rocker mechanisms) can be designed to satisfy this requirement. Grashof's rule states that if the sum of the shortest and longest link is less than that of the other two, and the shortest link is the crank, then a crank-rocker mechanism results. The students might be assigned the problem of finding a plane four-bar-linkage with the desired maximum output angle. Their solution might then be evaluated in terms of the comparative velocity and acceleration characteristics as well as the feasibility of building a linkage in the space available for it.

2. Appendix A shows how the author rearranged the equations shown on Exhibit 1 to to facilitate computation. This could be used to illustrate the technique of solving an implicit trigonometric equation in which a trigonometric function of the dependent variable appears on both sides of the equation in non factorable form.

3. For those groups which have had some experience in writing computational programs this case history provides an opportunity to write such a program. Appendix B shows a program written in Fortran IV for IBM System 360. This program is based on the equations in Appendix A and it computes values of displacement, velocity, and acceleration for a wide range of linkage parameters. A typical print-out is included in Appendix B.

4. The curves of displacement, velocity, and acceleration present an opportunity to use the techniques of graphical differentiation or integration. A worthy exericse, for example would call for verification of the velocity and acceleration curves - given the displacement curve.

5. A photograph of the production fixture used by Mark Systems is included in Appendix C to demonstrate the simple, low cost technique which was used to manufacture the actual links. While no actual data are available, the students might be asked to evaluate this approach for a production lot of one thousand. They may also be asked to present alternative methods for the same quantity and/or quantities of ten thousand and one hundred thousand.

References

R. S. Hartenberg and J. Denavit, Kinematic Synthesis of Linkages, McGraw Hill, 1964.

R. T. Hinkle, Kinematics of Machines, Prentice Hall, 1960.

K. H. Hunt, Mechanisms and Motion, John Wiley, 1959.

F. R. E. Crossley, "3-D Mechanisms", Machine Design, 1955.

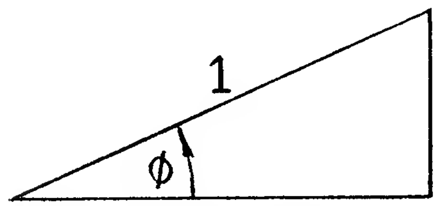
L. Harrisberger, "Technique for Synthesizing Space Crank Mechanisms", Machine Design, 1964.

J. O. Predale and A. B. Hulse Jr., "Introducing the Space Crank", Product Engineering, 1959.

DISPLACEMENT EQUATION

$$\cos\phi = \tan\gamma\cos\theta\ \sin\phi - \frac{\cos\lambda}{\cos\gamma}$$

$$\cos\phi = a\cos\theta\ \sin\phi - b$$



$$\left[1-\cos^2\phi\right]^{1/2} = \sin\phi$$

$$\cos\phi = a\cos\theta\sin\phi - b$$

$$\cos\phi = a\cos\theta\left[1-\cos^2\phi\right]^{1/2} - b$$

$$(\cos\phi + b)^2 = a^2\cos^2\theta\left[1-\cos^2\phi\right]$$

$$\cos^2\phi + 2b\cos\phi + b^2 = a^2\cos^2\theta - a^2\cos^2\theta\cos^2\phi$$

$$\cos^2\phi\left(1+a^2\cos^2\theta\right) + 2b\cos\phi + \left[b^2 - a^2\cos^2\theta\right] = 0$$

$$\cos^2\phi + \left[\frac{2b}{1+a^2\cos^2\theta}\right]\cos\phi + \left[\frac{b^2-a^2\cos^2\theta}{1+a^2\cos^2\theta}\right] = 0$$

$$\cos\phi = \frac{-\left[\frac{2b}{1+a^2\cos^2\theta}\right] \pm \sqrt{\left[\frac{2b}{1+a^2\cos^2\theta}\right]^2 - 4\left[\frac{b^2-a^2\cos^2\theta}{1+a^2\cos^2\theta}\right]}}{2}$$

$$\cos\phi = -\left[\frac{b}{1+a^2\cos^2\theta}\right] \pm \sqrt{\left[\frac{b}{1+a^2\cos^2\theta}\right]^2 - \left[\frac{b^2-a^2\cos^2\theta}{1+a^2\cos^2\theta}\right]}$$

VELOCITY EQUATION

$$\frac{\omega_o}{\omega_i} = \left(\frac{a \sin\theta}{1 + a\cos\theta \ \cot\phi}\right)$$

for $\omega_i = 1$ rad/sec : $\omega_{o_1} = \left(\frac{a \sin}{1 + a\cos\theta \ \cot\ \phi}\right)$

or $\omega_o = \omega_i \times \omega_{o_1}$

ACCELERATION EQUATION

$$\alpha_o = \frac{\left(\omega_i^2 + \omega_o^2\right) a\cos\theta + \omega_o \cot\phi \left(2\, a\, \omega_i \sin\theta - \omega_o\right)}{1 + a\cos\theta\cot\phi}$$

$$\alpha_o = \frac{\left(\omega_i^2 + \omega_i^2\ \omega_{o_1}^2\right) a\cos\theta + \omega_i\, \omega_{o_1} \cot\phi \left(2a\, \omega_i \sin\theta - \omega_i\, \omega_{o_1}\right)}{1 + a\cos\theta\ \cot\phi}$$

$$= \frac{\omega_i^2 \left(1 + \omega_{o_1}^2\right) a\cos\theta + \omega_i^2\ \omega_{o_1} \cot\phi \left(2a \sin\theta - \omega_{o_1}\right)}{1 + a\cos\theta\ \cot\phi}$$

$$= \omega_i^2 \left[\frac{\left(1 + \omega_{o_1}^2\right) a\cos\theta + \omega_{o_1} \cot\phi \left(2a \sin\theta - \omega_{o_1}\right)}{1 + a\cos\theta\cot\phi}\right]$$

```
LEVEL 13  (23 MAY 67)                    OS/360  FORTRAN H                        DATE  68.009/10.06.02

        COMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=50,SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID

                 C
                 C     PROGRAM CRANK
                 C
   ISN 0002            DIMENSION MAMMA(15),GAMMR(15),LAMDA(35),AMOR(35),AAMDA(35)
   ISN 0003        100 FORMAT (1H1,//,4X,'GAMMA=',I3,' DEG   LAMDA=',I4,' DEG   TAN(GAMMA
                      1)=',F6.4,'=A   COS(LAMDA)/COS(GAMMA)=',F6.4,'=B   OMEGA IN=1 RAD/S
                      2EC   ALPHA IN=0')
   ISN 0004        101 FORMAT (/,5X,'THETA,DEG   THETA,RAD     PHI,DEG     PHI,RAD    OME
                      1GA(OUT)  ALPHA(OUT)',/,5X,6('---------',3X))
   ISN 0005        102 FORMAT (5X,F6.1,F13.4,F14.4,F12.4,F12.4,F12.4)
   ISN 0006            ATOR=.0174533
   ISN 0007            DO 1 I=1,11
   ISN 0008            MAMMA(I)=10+5*(I)
   ISN 0009            GAMMR(I)=ATOR*MAMMA(I)
   ISN 0010            A=TAN(GAMMR(I))
   ISN 0011            JMIN = 5+I
   ISN 0012            JMAX = 33-I
   ISN 0013            DO 2 J=JMIN,JMAX
   ISN 0014            SN = 1.
   ISN 0015            LAMDA(J) = 5*J
   ISN 0016            AAMDA(J) = LAMDA(J)
   ISN 0017            AMDR(J)=ATOR*AAMDA(J)
   ISN 0018            B=COS(AMOR(J))/COS(GAMMR(I))
   ISN 0019            K=0
   ISN 0020            LMAX = 36
   ISN 0021            THETA = 0.
   ISN 0022          3 WRITE (6,100) MAMMA(I),LAMDA(J),A,B
   ISN 0023            WRITE (6,101)
   ISN 0024            DO 4 L=1,LMAX
   ISN 0025            THETR=ATOR*THETA
   ISN 0026            D=1.+(A*COS(THETR))**2
   ISN 0027            WWW=-B/D+SQRT((B/D)**2-(B**2-(A*COS(THETR))**2)/D)*SN
   ISN 0028            PHI = ARCOS(WWW)
   ISN 0029            PHIA=PHI/ATOR
   ISN 0030            DW=1.+A*COS(THETR)*COTAN(PHI)
   ISN 0031            WOWI=A*SIN(THETR)/DW
   ISN 0032            ALPHA=((1.+WOWI**2)*A*COS(THETR)+WOWI*COTAN(PHI)*(2.*A*SIN(THETR)-
                      1WOWI))/DW
   ISN 0033            WRITE (6,102) THETA,THETR,PHIA,PHI,WOWI,ALPHA
   ISN 0034          4 THETA = THETA + 2.5
   ISN 0035            K=K+1
   ISN 0036            IF(K.GT.1) GO TO 2
   ISN 0038            LMAX = 37
   ISN 0039            SN = -1.
   ISN 0040            GO TO 3
   ISN 0041          2 CONTINUE
   ISN 0042          1  CONTINUE
   ISN 0043            STOP
   ISN 0044            END
```

GAMMA= 15 DEG   LAMDA= 30 DEG   TAN(GAMMA)=0.2679=A   COS(LAMDA)/COS(GAMMA)=0.8966=B   OMEGA IN=1 RAD/SEC   ALPHA IN=0

THETA,DEG	THETA,RAD	PHI,DEG	PHI,RAD	OMEGA(OUT)	ALPHA(OUT)
0.0	0.0	134.9999	2.3562	0.0	0.3660
2.5	0.0436	135.0199	2.3565	0.0160	0.3656
5.0	0.0873	135.0796	2.3576	0.0319	0.3642
7.5	0.1309	135.1792	2.3593	0.0477	0.3620
10.0	0.1745	135.3182	2.3617	0.0635	0.3588
12.5	0.2182	135.4964	2.3649	0.0790	0.3547
15.0	0.2618	135.7132	2.3686	0.0944	0.3496
17.5	0.3054	135.9681	2.3731	0.1095	0.3436
20.0	0.3491	136.2606	2.3782	0.1244	0.3366
22.5	0.3927	136.5898	2.3839	0.1389	0.3286
25.0	0.4363	136.9548	2.3903	0.1530	0.3196
27.5	0.4800	137.3544	2.3973	0.1668	0.3096
30.0	0.5236	137.7831	2.4049	0.1800	0.2985
32.5	0.5672	138.2542	2.4130	0.1923	0.2863
35.0	0.6109	138.7515	2.4217	0.2050	0.2732
37.5	0.6545	139.2786	2.4309	0.2166	0.2589
40.0	0.6981	139.8340	2.4406	0.2276	0.2436
42.5	0.7418	140.4159	2.4507	0.2379	0.2273
45.0	0.7854	141.0228	2.4613	0.2474	0.2100
47.5	0.8290	141.6524	2.4723	0.2562	0.1918
50.0	0.8727	142.3026	2.4837	0.2641	0.1727
52.5	0.9163	142.9722	2.4953	0.2712	0.1528
55.0	0.9599	143.6582	2.5073	0.2775	0.1322
57.5	1.0036	144.3585	2.5195	0.2828	0.1110
60.0	1.0472	145.0714	2.5320	0.2871	0.0894
62.5	1.0908	145.7936	2.5446	0.2906	0.0675
65.0	1.1345	146.5235	2.5573	0.2930	0.0454
67.5	1.1781	147.2581	2.5701	0.2945	0.0233
70.0	1.2217	147.9947	2.5830	0.2951	0.0014
72.5	1.2654	148.7321	2.5959	0.2946	-0.0201
75.0	1.3090	149.4676	2.6087	0.2933	-0.0411
77.5	1.3526	150.1972	2.6214	0.2911	-0.0613
80.0	1.3963	150.9225	2.6341	0.2880	-0.0805
82.5	1.4399	151.6359	2.6465	0.2841	-0.0987
85.0	1.4835	152.3385	2.6588	0.2794	-0.1156
87.5	1.5272	153.0330	2.6709	0.2740	-0.1312

GAMMA= 15 DEG   LAMDA= 30 DEG   TAN(GAMMA)=0.2679=A   COS(LAMDA)/COS(GAMMA)=0.8966=B   OMEGA IN=1 RAD/SEC   ALPHA IN=0

THETA,DEG	THETA,RAD	PHI,DEG	PHI,RAD	OMEGA(OUT)	ALPHA(OUT)
90.0	1.5708	153.7114	2.6828	0.2679	-0.1453
92.5	1.6144	154.3741	2.6943	0.2613	-0.1579
95.0	1.6581	155.0217	2.7056	0.2542	-0.1690
97.5	1.7017	155.6455	2.7165	0.2466	-0.1784
100.0	1.7453	156.2505	2.7271	0.2386	-0.1863
102.5	1.7890	156.8385	2.7373	0.2304	-0.1926
105.0	1.8326	157.4022	2.7472	0.2219	-0.1975
107.5	1.8762	157.9465	2.7567	0.2132	-0.2010
110.0	1.9199	158.4687	2.7658	0.2043	-0.2032
112.5	1.9635	158.9675	2.7745	0.1954	-0.2043
115.0	2.0071	159.4453	2.7828	0.1865	-0.2043
117.5	2.0508	159.9006	2.7908	0.1776	-0.2034
120.0	2.0944	160.3332	2.7983	0.1688	-0.2017
122.5	2.1380	160.7450	2.8055	0.1600	-0.1993
125.0	2.1817	161.1341	2.8123	0.1514	-0.1963
127.5	2.2253	161.5018	2.8187	0.1429	-0.1929
130.0	2.2689	161.8487	2.8248	0.1346	-0.1892
132.5	2.3126	162.1745	2.8305	0.1264	-0.1853
135.0	2.3562	162.4805	2.8358	0.1184	-0.1811
137.5	2.3998	162.7671	2.8408	0.1106	-0.1769
140.0	2.4435	163.0336	2.8455	0.1030	-0.1727
142.5	2.4871	163.2820	2.8498	0.0955	-0.1686
145.0	2.5307	163.5114	2.8538	0.0882	-0.1645
147.5	2.5744	163.7230	2.8575	0.0812	-0.1606
150.0	2.6180	163.9171	2.8609	0.0742	-0.1569
152.5	2.6616	164.0947	2.8640	0.0675	-0.1534
155.0	2.7053	164.2543	2.8668	0.0608	-0.1502
157.5	2.7489	164.3984	2.8693	0.0544	-0.1472
160.0	2.7925	164.5267	2.8715	0.0480	-0.1444
162.5	2.8362	164.6389	2.8735	0.0417	-0.1420
165.0	2.8798	164.7351	2.8752	0.0356	-0.1399
167.5	2.9234	164.8167	2.8766	0.0295	-0.1381
170.0	2.9671	164.8829	2.8778	0.0235	-0.1366
172.5	3.0107	164.9346	2.8787	0.0176	-0.1355
175.0	3.0543	164.9710	2.8793	0.0117	-0.1346
177.5	3.0980	164.9927	2.8797	0.0058	-0.1341
180.0	3.1416	165.0002	2.8798	-0.0000	-0.1340